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Market Pricing of Deposit Insurance

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Abstract

We provide an approach to the market valuation of deposit insurance that is based on reduced-form methods for the pricing of fixed-income securities under default risk. By reference to bank debt prices as well as qualitative-response models of the probability of bank failure, we suggest how a risk-neutral valuation model for deposit insurance can be applied both to the calculation of fair-market deposit insurance premia and to the valuation of long-term claims against the insurer.

Key words: Deposit insurance pricing, risk-neutral default probability, bank failure.

1. Introduction

This paper proposes a methodology for, and an empirical implementation of, the market valuation of deposit insurance. We extend conventional reduced-form methods for the pricing of fixed-income securities under default risk in order to treat banks without public debt pricing. The resulting valuation model for deposit insurance is applied both to the calculation of fair-market deposit insurance premia and to the valuation of long-term claims against the insurer.

This paper is organized as follows. After reviewing the standard deposit insurance contract provided by the United States Federal Deposit Insurance Corporation (FDIC), section 2 provides a simplified formula for fair-market deposit insurance premia. By ''fair market,'' we mean the premia that would apply to insurance contracts that have zero net present market value when traded on public capital markets. We take no stand here on the policy issue of assignment of premia by the FDIC or other governmental insurers. Roughly speaking, once having adjusted probability assessments to incorporate the effects of investor risk premia, the fair-market deposit insurance premium is the probability of bank failure multiplied by the expected loss to the FDIC at failure per dollar of assessed deposits. Equivalently, the fair-market deposit insurance premium is the bank's short-maturity credit spread multiplied by the ratio of (a) the expected loss to the insurer given failure, per dollar of assessed deposits, to (b) the expected fractional loss given default on the bank's debt.

Section 3 proposes a more elaborate model able to treat the valuation of long-horizon insurance liabilities, in the spirit of modern models of the term structures of defaultable yields. Section 4 implements the model empirically, using a logit model to estimate the probability of bank failure, then applying a risk premium to convert actual to ''risk-neutral'' default probabilities. Section 5 applies the model to a selection of actual banks. Section 6 illustrates the implications of such models for the market value of claims against the insurer over long time periods, allowing for correlation between the term structure of interest rates, deposit growth, and bank failure risk. This is especially relevant if the actual insurance rates assessed by the FDIC remain significantly below the market-implied rates, for the analysis suggests some of the key parameters and risk factors determining the initial capital that would be held by the FDIC in order to offset the present market value of these liabilities.

2. Fair-market premia—an overview

This section provides an overview of the approach, and simple approximate fair-market premia calculations.

2.1. The terms of the FDIC contract

Figure 1 illustrates the basic FDIC deposit insurance contract. At the beginning of each quarter, from each surviving insured financial institution (''bank''), the FDIC collects a deposit insurance premium equal to the product of the premium rate associated with the bank's risk classification and the quantity of assessed deposits at the bank, as measured at the end of the previous quarter. Because the FDIC assigns risk classifications only twice a

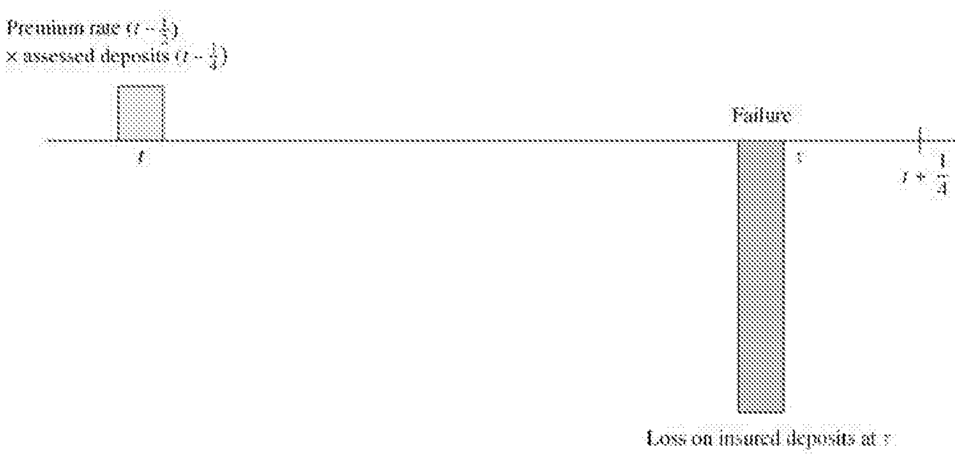


Figure 1. **FDIC cashflows, ''even'' contract periods.**

Table 1. **FDIC risk classifications and insurance premia (in bp)**

Capital Group	A (bp)	B (bp)	C (bp)
1. Well capitalized	0	3	17
2. Adequately capitalized	3	10	24
3. Undercapitalized	10	24	27

year, these classifications lag current conditions by one quarter in odd quarters, and by two quarters in even quarters, as illustrated in figure 1. In the event that a bank fails, the FDIC covers losses on any insured deposits that cannot be recovered from the failing bank's assets, to a maximum of $100,000 per insured account. The resolution process is summarized in FDIC (2002). The risk classifications, and the current associated annualized premium rates per dollar of assessed deposits, are shown in table 1. The classification is by capitalization groups: (1) well capitalized, (2) adequately capitalized, and (3) undercapitalized, according to criteria for various accounting-based capital ratios. The second factor of the risk classification is determined by a supervisory subgroup, A, B, or C, which is based on general financial soundness, as measured by CAMEL ratings that are not publicly observable. One may obtain details at the FDIC Internet site.[1] The 1A risk classification, associated with a deposit insurance premium of zero, accounts for 92.5% of financial institutions, and 96.7% of deposits insured in the FDIC system, according to FDIC data[2] for 2002.

Notably, the quantity of assessed deposits differs from the quantity of deposits that are insured as of the time of failure, because of the lag in time between assessment and failure, during which there can be withdrawals and new deposits, and also because the assessed measure of deposits does not reflect the insurance cap of $100,000 per account. Based on the FRB call-report database, we found that the mean and median ratios of uninsured to insured deposits, across all U.S. commercial banks during the period 1976–2000, are 11.39% and 8.80%, respectively. The expected time lag between assessment and failure is roughly 1.5 quarters, provided failure times may be reasonably approximated as uniformly distributed within quarters. Fluctuations in deposit levels within this lag period could be important if there is a sufficiently strong moral-hazard incentive for riskier banks to increase leverage by raising deposits once its deposit insurance premia for the subsequent quarter are determined. Presumably, as depositors become aware of significant failure risk at a bank, they reduce their levels of uninsured deposits, so one would expect this ratio to be much smaller among failing banks.

2.2. Prior literature

The previous literature estimating fair-market FDIC insurance premia has exclusively adopted a structural approach, which treats default as the event that the market value of the

1 See www.fdic.gov/about/learn/symbol/index.html

2 See the memorandum to the FDIC Board by FDIC Director Arthur Merton at www.fdic.gov/deposit/insurance/bif200202.html

bank's assets, typically modeled as a diffusion process, is insufficient to meet its liabilities (see Merton, 1977; Marcus and Shaked, 1984; Ronn and Verma, 1986; Pennacchi, 1987; Duan et al., 1995; Epps et al., 1996; Duan and Yu, 1999). As opposed to this endogenous default-time modeling approach, the reduced-form approach, taken in this paper, treats default as a stopping time whose arrival intensity may depend, in an exogenously parametrized fashion, on such covariates as leverage, credit rating, or macroeconomic conditions.

Although conceptually useful, the structural approach is hampered by the difficulty with which the form of the FDIC insurance contract can be consistently captured within the modeled capital structure of the bank. For this, one must assume a particular liability and auditing structure of the bank, so that bankruptcy can be determined endogenously. Structural models typically base asset values and asset volatility on equity prices, but neglect the effect of the deposit insurance contract itself. As pointed out by Falkenheim and Pennacchi (2002), this approach is problematic for banks that are not publicly traded. In contrast, the reduced-form approach need not treat the exact form of the insurance contract because it does not depend on the liability structure or auditing process of the bank. While this means that reduced-form models could fail to capture some potentially important sources of endogeneity, they do allow for simpler and, in our view, more robust estimates of market insurance premia in practical settings.

As we shall see, short-term fair-market deposit insurance premia are, roughly speaking, proportional to credit spreads at maturities of less than six months. Reasonable estimates of fair-market deposit insurance premia therefore call for a model whose implied short-term credit spreads are reasonable. The credit spreads at maturities under one year that are implied by structural models, however, are highly sensitive to the assumed stochastic properties of the asset process. For example, typical structural models, such as those cited, take the asset process to be a diffusion. This dramatically understates credit spreads at short maturities. That is, once such a model is calibrated to the leverage and asset volatility of even a relatively risky issuer, the model-implied risk-neutral probability of default within a few months is essentially negligible. (Duffie and Lando, 2001 and Collin-Dufresne and Goldstein, 2001 provide examples of this distortionary effect on short-term credit spreads.) More reasonable short-term spreads can, in principle, be obtained in structural models that allow surprise downward jumps in assets, as in Zhou (2001), or allow for imperfectly observed assets, as in Duffie and Lando (2001). It may be difficult to calibrate such models, however, given the limited availability of data on asset jumpiness or accounting quality. (Yu, 2002, provides some empirical estimates of the effect of accounting quality on short-term credit spreads.)

2.3. A simple robust formula for insurance premia

For the purpose of obtaining a simple formula for fair-market insurance premia, we begin with an extremely basic model.

The ''risk-neutral'' pricing approach that we take is based on the fact that, in the absence of capital-market frictions and arbitrage opportunities, there exist altered probability assessments, called ''risk-neutral probabilities,'' with the property that a claim to a risky cash flow of C at a future time t has an initial market value equal to its discounted expected cash flow,

$$z(t)E^*(C), \tag{1}$$

where $z(t)$ is the discount on risk-free cash flows received at time t, and E^* denotes expectation with respect to risk-neutral probabilities. This general approach to contingent claims pricing is due to Cox and Ross (1976) and Harrison and Kreps (1979). For discrete time periods, the existence of such risk-neutral probabilities is shown by Dalang et al., (1990). This particular variant of risk-neutral probabilities is based on a concept of Jarrow (1987) that has come to be known as "forward risk-neutral measure."

For our simple illustrative model, which will be extended in the next section, we take contract time periods of some relatively short duration t, and suppose for simplicity that the insurance claim is payable at the end of the period. Per dollar of assessed deposits, the contingent claim against the FDIC is $C = 1_{\tau<t}\, L$, where $1_{\tau<t}$ is the indicator that the bank's failure time τ is before t (having an outcome of 1 if default before t, zero otherwise), and L is the loss claim, net of recoveries, per dollar of assessed deposits. For this simple model, we will suppose that L has a risk-neutral mean ℓ that does not depend on the contract period t. This assumption may be reasonable for short time periods such as one quarter or less. The risk-neutral mean loss per dollar of assessed deposits is

$$E^*(1_{\tau<t}L) = E^*(1_{\tau<t}E^*(L \mid \tau<t)) = q(t)\ell,$$

where $q(t) = E^*(1_{\tau<t})$ is the risk-neutral probability of default before t. The annualized fair-market deposit insurance premium per dollar of assessed deposits is thus

$$V(t) = \frac{1}{t}q(t)z(t)\ell, \tag{2}$$

having multiplied by t^{-1} in order to annualize. Both $q(t)$ and ℓ reflect risk premia that investors in deposit insurance contracts would demand for bearing the risk of default timing and the risk of uncertain losses given default. Typically, uncertainty in the timing of bank failure carries a positive risk premium because the incidence of bank failure is positively correlated with poor macroeconomic conditions. In this case, the risk-neutral default probability $q(t)$ would be larger than its actual counterpart, $p(t) = E(1_{\tau<t})$. In section 3, we propose an upward scaling of statistical estimates of the actual default probability $p(t)$ so as to estimate the risk-neutral default probability $q(t)$.

In the limit, as the contract length t becomes small (assuming differentiability), the risk-free discount $z(t)$ goes to 1 and the annualized risk-neutral default probability $q(t)/t$ converges[3] to the initial risk-neutral mean failure arrival rate $\lambda^*(0)$. From (2), the limiting "short-period" annualized fair-market deposit insurance premium per dollar of assessed deposits is

3 As t changes, so does the risk-neutral forward measure, but the limit is in any case the initial default intensity $\lambda^*(0)$ under the "usual" risk-neutral measure, defined in section 3. For regularity, it is enough that default intensity with respect to the usual risk-neutral measure exists, and that the short-interest-rate process and the usual risk-neutral intensity process are bounded. Weaker conditions would suffice.

$$\rho(0) = \lambda^*(0)\ell. \tag{3}$$

As we shall later discuss, this ''short-term'' premium is a reasonable approximation of the quarterly fair-market deposit premium $\rho(0.25)$, provided the loss parameters, $\lambda^*(0)$ and ℓ, are themselves reasonable.

To illustrate, suppose that, under risk-neutral probabilities, a bank has an annual mean failure rate of $\lambda^*(0) = 2\%$, and that the FDIC expects to lose 10 cents per dollar of assessed deposits. The associated ''short-term'' fair-market deposit insurance rate is then 20 basis points, on an annualized basis. On an assessed deposit base of, say, \$100 million, the quarterly deposit insurance payment by the bank would be $0.25 \times 0.20\% \times \$100{,}000{,}000 = \$50{,}000$.

By the same reasoning and assumptions, the short-term market credit spread on the same bank's debt is $S(0) = \lambda^*(0)\ell_D$, where ℓ_D is the risk-neutral expected fraction of the face value of the debt lost at default. Thus, substituting $\lambda^*(0) = S(0)/\ell_D$ into (3), we can also express the short-term fair-market deposit insurance premium as

$$\rho(0) = S(0)\frac{\ell}{\ell_D}. \tag{4}$$

For illustration, suppose that a bank borrows at a short-term credit spread of $S(0) = 100$ basis points, and that, under risk-neutral probabilities, lenders expect to lose $\ell_D = 50$ cents per dollar of principle at default, while the FDIC expects to lose $\ell = 10$ cents per dollar of assessed deposits. Then the associated annualized ''short-term'' fair-market deposit insurance premium is again 20 basis points.

One quarter is not an ''instantaneous'' period. We have done robustness checks based on the quarterly model provided in section 6, which accounts for the timing of payments within the quarter and for random fluctuation within the quarter of interest rates, the insured quantity of deposits, and default risk, allowing for correlation among these variables. Letting $S(0.25)$ denote the credit spread at a maturity of one quarter, we find that

$$\rho(0.25) \simeq S(0.25)\frac{\ell}{\ell_D}, \tag{5}$$

is a highly accurate approximation of the model-implied quarterly fair-market deposit insurance premium $\rho(0.25)$, across a wide range of parameters. A weakness of this simple modeling approach is not the accuracy of the simple approximation (5) itself, but rather the difficulty with which the inputs $S(0.25)$, ℓ, and ℓ_D can be estimated.

Indeed, credit spreads on short-term bank debt, even when available, tend to be contaminated by the effects of liquidity. ''Default-free'' benchmark interest rates, from which one estimates credit spreads, are also difficult to measure, because of the influence of tax abatements, ''moneyness,'' and repo effects on treasury rates. General collateral rates are preferable as a benchmark, and one could also approximate by using LIBOR rates as a benchmark, although LIBOR rates themselves reflect some credit risk. An advantage of LIBOR rates is that the effects of liquidity on bank debt spreads are to some extent reduced when measured relative to LIBOR rates. One might also consider, however, an add-on to the modeled market deposit-insurance premia in order to compensate investors for illiquidity in the market for deposit insurance contracts.

Table 2. **FDIC premia and market credit spreads (in bp)**
The spreads are based on Datastream and Bloomberg data, and are relative to LIBOR swap rates of comparable maturities, as of September, 2002. Default swap (CDS) rates were obtained from CreditMetrics on the same date.

	Estimated Risk Class	Implied FDIC Premium	1-Year Spread	5-Year[b] Spread	5-Year CDS Rate
JPMorgan-Chase	1A	0	43	70	80
Citigroup	1A	0	50	76	60
Bank One	1A	0	49	36	43
Bank of America	1A	0	44	63	48
Wachovia	1A	0	54	90	—
Mellon Bank	1A	0	54	71	—
Union Planters Corp.	1A	0	83	—	—
Hudson United Bancorp.	1A	0	250	—	—
Provident Bank	1A	0	90	—	—
PNC Bank	1B	3	74	—	—
Sovereign Bancorp Inc.	—	—	350	—	—

While there are some studies of bank debt losses given default on bank debt, such as Ammer and Packer (2000), we are not aware of models or published statistics concerning the losses to the FDIC at bank failure per dollar of assessed deposits. Our enquiries suggest that the FDIC, which publicly reports statistics on losses to the FDIC as a fraction of bank assets, may also be in a position to obtain at least actuarial estimates of losses at failure per dollar of assessed deposits from its database of recent bank failures. Such estimates could control for bank size, risk classification, and other variables that are available at the time that deposit insurance premia are assessed.

Table 2 provides credit spreads for a selection of publicly traded banks. The one-year and five-year spreads are relative to similar-maturity LIBOR swap rates, as of September, 2002, based on Bloomberg data. The reported default swap (CDS) rates were supplied to us, via CreditMetrics, from a market vendor of default swaps, which are essentially default insurance contracts on debt. It can be shown[4] that, in frictionless markets, CDS rates are approximately equal to par credit spreads. The estimated risk classifications shown in table 2 are based on ratings supplied by bank rating agencies such as Bauer Financial, and by the CRA ratings of FFIEC.[5] The actual FDIC deposit insurance premia associated with the estimated risk classifications are also shown, for a selection of banks, in table 2, for comparison with the credit spreads. Section 5 provides a comparison of modeled fair-market deposit insurance premia and the premia assessed by the FDIC.

Referring to (4), it is apparent from table 2 that current FDIC deposit insurance premia are far lower than fair-market deposit insurance premia, unless one assumes that the losses to the FDIC given failure are negligible compared to bank debt losses given default. One hypothesis for the relatively low deposit insurance premia currently assessed by the FDIC

4 See Duffie (1999).
5 See www.fdic.gov/bank/individual/bank/.

is that these premia are based on an actuarial (that is, average-historical-loss) approach that does not attempt to account for market risk premia. We know that fair-market deposit insurance premia would be elevated over actuarial mean-loss rates in order to compensate investors for bearing default risk. For example, investment-grade banks (those rated Baa to Aaa by Moody's) have an average annual default rate for 1983–1998 of 0.0%, according to Ammer and Packer (2000), whereas bank credit spreads for A-rated banks, even when measured relative to LIBOR (and thus stripping out some of effects of illiquidity and LIBOR credit risk), are currently on the order of 30–50 basis points. Even on the basis of a structural model of deposit insurance pricing, Pennacchi (1999) also finds that fair-market deposit insurance premiums would be significantly greater than actuarial mean-loss rates, and that the difference between the two rises with the credit risk of the bank.

Estimating risk premia associated with losses given default is problematic, although this difficulty is to some extent mitigated by the fact that it is the ratio of ℓ to ℓ_D that determines the fair-market insurance premium in (4), and loss-given-default risk premia would presumably factor up both of the risk-neutral means, ℓ and ℓ_D, from their respective actual means by similar and thus partially offsetting amounts.

The three-month market credit spread $S(0.25)$ is not directly observable for most banks, although spreads on private lending to banks may be observable and may, if lending to banks is reasonably competitive, be representative of what their market credit spreads would be. In some cases, the market credit spread of a given bank may be approximated from the market spreads of comparable banks, after controlling for bank size, credit rating, and other observable variables.

We also pursue in the following section an approach based on the more basic formula (3). Rather than estimating the risk-neutral mean default arrival intensity λ^* directly from the target bank's credit spreads, we propose in section 3 to approximate λ^* with $\lambda\pi$, where λ is that bank's actual (not risk-neutral) mean default arrival rate, estimated using a conventional logit model, and π is a scaling factor that approximates for the impact of default risk premia. We propose to obtain an estimate of the risk-premium scaling factor π from those banks for which we can estimate both the risk-neutral default intensity $\lambda^* = S/\ell_D$ (from market credit spreads and default recovery data), and the actual intensity λ, estimated from a logit model. That is, one could estimate $\pi = \lambda^*/\lambda$ from banks with observable credit spreads, and apply this risk-premium factor to get an estimate of the risk-neutral default intensity of banks without observable credit spreads.

3. Fair-market deposit insurance premia

This section lays out an analytically tractable reduced-form model for the market valuation of deposit insurance liabilities. Subsequent sections estimate and illustrate special cases of the model. We propose a multi-factor state process of the dynamics of interest rates, the given bank's failure risk, and the bank's deposit growth.

Following the standard paradigm of Harrison and Pliska (1981), we fix some probability space and information filtration. The short-term benchmark interest-rate process r is given. This means that one may invest one dollar in risk-free deposits at any given time s and continually re-invest interest payments so as to have a market value of $e^{\int_s^t r(u)\mathrm{d}u}$ dollars by any future time t.

Assuming frictionless markets and no arbitrage, theory tells us that, under technical regularity conditions, risk-neutral probabilities can be chosen with the property that any traded contingent claim promising a payment of Y at time t has a market value at any time s before t of $E_s^*(e^{\int_s^t -r(u)\mathrm{d}u}Y)$, where E_s^* denotes conditional expectation at time s, given all public information, with respect to these risk-neutral probabilities. This is the usual starting point for risk-neutral derivative pricing models.[6]

Fixing a hypothetical bank, we suppose that there is a multi-dimensional state process X determining interest rates, risk-neutral default intensities, and deposit growth. We assume, moreover, that, under our risk-neutral probabilities, X is an affine process, meaning that the logarithm of the characteristic function of $X(t)$ given $X(s)$ is affine (constant plus linear) with respect to $X(s)$ (e.g., Duffie et al., 2000). Specific multivariate diffusion examples of X will be spelt out shortly. This framework also allows for jumps in the affine state process. Our interest rate model is $r_t = R(X_t)$, where $R(\cdot)$ is affine. The risk-neutral bank default intensity model is $\lambda_t^* = \Lambda(X_t)$, where $\Lambda(\cdot)$ is also affine. This allows for correlation between bank failure and the level and slope of the term structure of interest rates, which is relevant for banks with an asset–liability duration mismatch. Finally, letting D_t denoting assessed deposits at time t, the assumed deposit-growth model is $D_t = e^{d(X(t))}$, where $d(\cdot)$ is affine. This allows for dependence of deposit growth on market interest rates and on the bank's failure risk, as well as some momentum effects in deposit growth. The implications of these correlation effects for the market valuation of long-horizon insurance liabilities are explored in section 6.

As in section 2, we suppose that, in risk-neutral expectation, a fraction ℓ of the most recently assessed quantity of deposits is the net claim on the FDIC at failure. The model of default arrival is doubly stochastic, driven by X, as defined, for instance, by Karr (1991). This means that, conditional on the path X of the state process, default occurs as the first arrival of a Poisson process with (conditionally deterministic) risk-neutral intensity $\lambda_t^* = \Lambda(X_t)$. In particular, conditional on the path of X, the risk-neutral probability of survival of the bank to time t is $e^{-\int_0^t \lambda^*(u)\mathrm{d}u}$.

The total market value of the FDIC insurance claims liabilities for this bank between time 0 and some future time T is

$$v(T) = \int_0^T \phi(t)\ \mathrm{d}t, \tag{6}$$

where $\phi(t)\mathrm{d}t$ is the market value of depositors' claims on the FDIC due to failure ''at'' time t, in the sense of a density. That is,

$$\phi(t) = E^*\left[e^{-\int_0^t r(u)\mathrm{d}u} e^{-\int_0^t \lambda^*(u)\mathrm{d}u} \lambda^*(t)\ell D_t\right]. \tag{7}$$

Inside the expectation in (7), after conditioning on the path X of the state process, one finds

6 This notion of risk-neutral probabilities differs from the notion of forward risk-neutral measure used to illustrate the basic idea in section 2.

a discount factor $e^{-\int_0^t r(u)\mathrm{d}u}$ for interest rates, the conditional risk-neutral probability density $e^{-\int_0^t \lambda^*(u)\mathrm{d}u}\lambda^*(t)$ of the default time τ, and the risk-neutral conditional expected loss given default at t, ℓD_t, given information just before t. From the properties of affine processes and (7),

$$\phi(t) = e^{\alpha(t)+\beta(t)\cdot X_0}(\gamma(t) + \nu(t)\cdot X_0), \tag{8}$$

where, as usual for affine models, the coefficients $\alpha(t)$, $\beta(t)$, $\gamma(t)$, and $\nu(t)$ can be calculated analytically, by solving certain ordinary differential, generalized Riccati, equations, as explained in Duffie et al. (2000).

The total market value $v(T)$ of the insurance liability between time 0 and time T can then be computed from (6) by numerical integration, as is done in our results to follow.

4. An empirical implementation

This section provides an empirical implementation of the preceding affine framework for deposit insurance pricing. Because bank risks are classified by the FDIC twice a year, we will treat the insurance contract period as six months, with an automatic renewal at three months at the same premium rate, given survival to that midpoint. For a contract period of six months or less, as our robustness checks have shown, the details of the specification of interest-rate behavior, deposit growth, and failure risk are not crucial. Indeed, we will treat the risk-neutral failure intensity as constant within the six-month period until default.

4.1. Time-series specification

The underlying affine state process X is taken to be $X(t) = (r(t), G(t))$, where $r(t)$ is the short-term interest rate and $G(t)$ is a measure of GDP growth. We model X as an affine diffusion process driven by a risk-neutral two-dimensional standard Brownian motion B^*. Specifically,

$$\mathrm{d}r_t = a[\bar{r}(t) - r_t]\mathrm{d}t + \sigma_r \mathrm{d}B^*_{1t}, \tag{9}$$

where a and σ_τ are positive constants, and $\bar{r}(t)$ chosen to match the initial forward rate curve.[7] This has been called an extended Vasicek model.

As a proxy for gross domestic product (GDP) growth, we actually chose to measure the component of GDP growth that is orthogonal to interest rate changes, and so chose to model $G(t)$ as

$$\mathrm{d}G_t = \sigma_G \mathrm{d}B^*_2(t), \quad G_0 = 0, \tag{10}$$

where σ_G is a constant. The level of assessed deposits at time t is then assumed to be of the form

7 Letting $f(0,t)$ denote the instantaneous forward rate at time 0 for maturity t, and assuming differentiability, one calculates that $\bar{r}(t) = f(0,t) + [\partial f(0,t)/\partial t + \sigma_r^2(1 - e^{-2at})/2a]/a$.

$$D_t = e^{\delta_0 + \delta_1 r(t) + \delta_2 G(t)}, \tag{11}$$

for constants δ_0, δ_1, and δ_2.

Assuming that the risk-neutral hazard rate λ^* is a constant up until the end of the insurance contract period, the present value of the FDIC insurance six-month liability can be calculated from (6) as

$$v(0.5) = \lambda^* \ell \int_0^{0.5} e^{-s\lambda^* + \eta(s)} ds, \tag{12}$$

where $\eta(s)$ is provided explicitly in the appendix.

We let ρ be the annualized insurance premium charged per dollar of assessed deposits. Given the quarterly lag in the insurance premium payments, the present value of the FDIC premiums received is

$$\frac{\rho}{4}[D_{-0.25} + D_0 e^{-0.25\lambda^*} z(0.25)], \tag{13}$$

where $z(0.5) = E^*(e^{-\int_0^{0.25} r(u) du})$ is the price of a zero-coupon default-free bond maturing in one quarter. This present value is the sum of the initial premium paid $D_{-0.25}\rho/4$ plus the present value of the premium paid in the subsequent quarter, which is paid if and only if the bank survives for another quarter, which happens with risk-neutral probability $e^{0.25\lambda^*}$.

Combining (12) and (13),

$$\rho = 4 \cdot \frac{\lambda^* \ell \int_0^{0.5} e^{-s\lambda^* + \eta(s)} ds}{D_{-0.25} + D_0 e^{-0.25\lambda^*} z(0.25)}. \tag{14}$$

In order to apply this formula, one needs estimates of the parameters $(\delta_0, \delta_1, \delta_2)$ of the deposit growth model, the parameters a and σ_r of the interest-rate process r, current short-term interest rates for maturities of up to six months, the volatility parameter σ_G of the GDP growth process, the risk-neutral mean loss rate ℓ per dollar of assessed deposits at failure, and the risk-neutral failure intensity λ^*. Estimates of λ^* could be based on short-term credit spreads, using $\lambda^* = S(0.5)/\ell_D$, where $S(0.5)$ is the spread at a maturity of six months and ℓ_D is the risk-neutral fractional loss given default on bank debt. In what follows, however, we will estimate λ^* as $\lambda\pi$, where λ is a logit estimate of the actuarial hazard rate and π is a risk-premium scaling factor.

4.2. Logit analysis of bank failure probabilities

This section estimates the actuarial bank failure hazard rate λ using historical data from the FDIC on bank failures. This FDIC database contains 1608 commercial bank failures during the period 1976–2000. The covariates for our logistic model are obtained from the Chicago Federal Reserve Bank database of bank Call Reports, which contains quarterly accounting information on all commercial banks. For example, in the first quarter of 2000, approximately 9000 banks were represented. Following Kashyap and Stein (2000), we adjust the call-report data in order to remove the known inconsistencies.

Table 3. **Deposits (in millions of dollars) at failed commercial banks**

Year	Number of Failures	Failed Banks Mean Deposit	Failed Banks Total Deposits	All Banks Total Deposits	Percent Failed Deposits
1976	17	72.65	1235.11	830,924.00	0.15
1977	6	31.10	204.58	929,169.00	0.02
1978	6	140.39	842.32	1,233,403.00	0.07
1979	10	11.08	110.75	1,362,804.00	0.01
1980	11	474.70	5221.65	1,481,163.00	0.35
1981	7	14.31	100.15	1,588,782.00	0.01
1982	35	62.47	2186.61	1,705,689.00	0.13
1983	46	79.88	3674.38	1,842,503.00	0.20
1984	79	395.65	31,256.01	1,962,827.39	1.59
1985	118	25.96	3063.22	2,118,087.86	0.14
1986	144	49.96	7194.14	2,283,527.39	0.32
1987	201	36.25	7287.02	2,335,455.89	0.31
1988	280	136.86	38,320.24	2,431,734.67	1.58
1989	206	113.96	23,475.51	2,548,504.78	0.92
1990	159	64.56	10,264.83	2,650,149.96	0.39
1991	108	341.67	36,900.54	2,687,663.57	1.37
1992	100	147.91	14,790.99	2,698,681.12	0.55
1993	42	69.88	2935.02	2,754,329.07	0.11
1994	11	81.08	891.87	2,874,438.54	0.03
1995	6	129.40	776.39	3,027,574.13	0.03
1996	5	39.53	197.65	3,197,135.79	0.01
1997	1	27.51	27.51	3,421,725.77	0.00
1998	3	86.89	260.68	3,681,442.85	0.01
1999	7	179.90	1259.27	3,830,826.41	0.03
Total	1608	119.70	192,476.43		0.35

Table 3 shows descriptive statistics[8] on the bank failures in our database. Given, by year, are the number of bank failures, the mean deposits of the failed banks, the total deposits of the failed banks, the total deposits of all banks, and the percentage of total deposits that the failed banks represent. For example, the largest annual fraction of deposits held at failed banks during this period is 1.59%, for 1984.

In order to estimate the actuarial hazard rate λ, we use a stepwise logistic regression, allowing additional explanatory variables to enter the regression model only if they are significant at the 5% confidence level and if, when included, all previously entered covariates remain significant at the 5% level. (This procedure is sensitive to the order in which new covariates are considered.) We estimate the logistic regression using the data

8 Table 3 presents the descriptive statistics of bank failures over the period 1976–1999. We produce results starting from 1976, as this is the first year from which we have Call-Report data. The table presents results based on only commercial bank failures. Savings-and-Loans institutions are not included. Failed banks' mean deposit is the average of the deposits outstanding (as of the quarter immediately preceding the failure date) of banks that failed in the given year. Similarly, Failed Banks' Total Deposits is the sum of the outstanding deposits of banks that failed in the given year.

Table 4. **Logit estimation of annualized bank failure intensity, from quarterly Call-Report data, including all failures, 1976–1999, for which data were available (1449 failures included from a universe of 1608 failures)**

Covariates: Size (logarithm of total bank assets), NI/TA (ratio of net income to total assets), D/E (ratio of total deposits to total equity), L/TE (ratio of loans to total assets), TA/TL (ratio of total assets to total liabilities), PL/TL (ratio of provisions for bad loans to total liabilities).

Panel A: Maximum likelihood estimates

Covariate	DF	Parameter Estimate	Standard Error	Wald's χ^2	p-Value	Odds Ratio
Intercept	1	55.9	1.06	2801.9	< 0.0001	
Size	1	− 0.410	0.0276	220.3	< 0.0001	0.664
NI/TA	1	− 8.33	2.08	16.0	< 0.0001	< 0.001
D/E	1	0.0001	0.0001	7.0	0.0082	1.000
L/TA	1	2.000	0.222	81.2	< 0.0001	7.387
TA/TL	1	− 55.4	0.934	3518.9	< 0.0001	< 0.001
PL/TL	1	− 7.82	2.25	12.1	0.0005	< 0.001

Panel B: Cross-sectional distribution of fitted hazard rates (in bp)

			Percentiles of Failure Hazard Rates				
Quarter	Number of Banks	Mean	10th	25th	50th	75th	90th
2000Q1	7489	15.3	0.14	1.49	7.44	20.5	38.7
2000Q2	7413	17.1	0.13	1.44	7.07	20.0	37.7
2000Q3	7302	13.1	0.10	1.21	6.18	17.4	32.9
2000Q4	7209	11.4	0.11	1.14	5.62	15.7	29.1

from the period[9] 1976–1999, and use the estimated model to predict failure rates for the four quarters of 2000. The resulting logistic regression model and the hazard-rate estimates for the four quarters of 2000 are shown in table 4.

The covariates are chosen based on the past literature and based on their predictive power in the maximum likelihood estimation. The logarithm of total assets is a size-related covariate. Profitability is measured as the ratio of net income to total assets (NI/TA). Leverage is proxied with the ratio of deposits to total equity (D/E). The composition of the asset portfolio is captured by the ratio of loans to total assets (L/TA). Another leverage measure that we considered is the ratio (TA/TL) of total assets to total liabilities (not including equity capital). We include a measure of the quality of the loan portfolio, the ratio of bad loans to total liabilities (PL/TL). Panel A of table 4 presents the results of the logistic regression. Panel B provides summary statistics of the predicted default rates for the four quarters of 2000. The rates are annual hazard rates.

The signs of the reported coefficients are as expected. A bank is estimated to be less likely to fail if it is larger, if it is more profitable, and if its asset–liability ratio is higher. Higher loan-loss reserves, relative to liabilities, implies lower default likelihood. Going

9 We used all bank failures during 1976–1999 for which enough accounting data was available in the call reports (1449 bank failures were used, out of a universe of 1608 bank failures).

the other way, failure likelihood is estimated to be increasing in the bank's deposits (relative to its equity capital) and increasing in the fraction of assets that are loans. From Panel B, one notes that the 10-percentile mean failure rate, 0.0014%, is far smaller than the 90-percentile mean failure rate, 0.39%.

4.3. Other parameter estimates

This subsection describes the remaining parameter estimates used in the fair-premium calculations.

4.3.1. Term-structure model. Daily observations of the term structure of interest rates from 1984–2001 were obtained from the Kamakura Corporation. Monthly observations of the one-month T-bill rate were obtained from Ken French's website. The parameters of the extended Vasicek model, ($a = 0.007200$, $\sigma_r = 0.010059$), were taken from Janosi et al. (1999).

4.3.2. Deposit model. Quarterly data on U.S. GDP were obtained from the Bureau of Economic Analysis. In constant 1996 dollars, the quarterly change in GDP was regressed on the current one-month T-bill rate, over the period 1947–2002. The residual from this regression is taken to be our orthogonalized GDP growth rate variable, G_t. The standard error of the regression determines our estimate for $\sigma_G^2 = 1732$.

For each bank for which at least eight quarters of data were available during 1976–2000, we estimated a deposit model under which the logarithm of the bank's insured deposits is regressed on the T-Bill rate and the orthogonalized GDP growth rate, using quarterly call-report data, for each commercial bank. The estimates, based on a total of 20,184 observations, are shown for selected quantiles in Panels A and B of table 5. Panel A presents the deposit regression results, for each of selected quantile. Panel B shows the cross-sectional distribution of the t-statistics of these estimates.

As shown, for the median bank, deposits are negatively related to interest rates and not significantly related to the orthogonalized GDP growth rates. For the reported fair-premium estimates to follow, we used the point estimates, bank by bank. Many alternative deposit models are conceivable; ours is not definitive (see Craig and Thomson, 2003).

4.3.3. Deposit loss and risk premium. Two of the most difficult parameters to estimate, given the available data and literature, are the loss rate ℓ on assessed deposits and the risk-premium scaling factor π. We report fair-market deposit premiums for ranges of these parameters, centered on values observed in the literature. For example, the average loss rate on assets as reported in the FDIC failed Bank Cost Analysis has fluctuated widely on a year-to-year basis, from a low of 8% for 1992 to a high of 61% for 1999. Loss rates also appear to be dependent on the size of the commercial bank's assets, the charter type of the bank, and the state of the bank's domicile. For this reason, we report results for a range of alternative loss-given-failure rates.

Estimates for the risk-premium scaling factor π are not readily available. A recent study by Chava and Jarrow (2002) showed large variations in this parameter. Based on the S&P

Table 5. **Cross-sectional distribution of estimated deposit models**

For each bank for which at least eight quarters of data are available, OLS estimates are obtained for the quarterly period model $\log(Y(t)) = l_0 + l_1 r(t) + l_2 X(t) + (t)$, where $Ye(t)$ is the insured deposits, $e(t)$ is the residual, $r(t)$ is the spot interest rate, $X(t)$ is the current residual of a regression of the quarterly change in GDP on the the spot interest rate. Panel A shows statistics of the cross-sectional distribution of the fitted coefficients and adjusted R^2 coefficients of the linear models. Panel B provides the cross-sectional distribution of the t-statistics of the coefficient estimates. The total number of observations (cross-sectional and time-series), is 20,184.

		Percentiles				
Variables	Mean	5th	25th	Median	75th	90th
Panel A: Parameter estimates						
Intercept	10.70	8.27	9.72	10.61	11.53	13.58
Spot rate	– 6.88	– 27.00	– 10.03	– 4.67	1.12	8.91
GDP	0.00074	– 0.00263	0.00000	0.00069	0.00138	0.00466
R^2	24.37%	1.67%	10.70%	21.85%	33.01%	60.38%
Panel B: t-statistics						
Spot rate	– 1.72	– 6.06	– 4.37	– 2.49	0.64	3.59
GDP	0.60	– 1.36	0.00	0.72	1.24	2.23

rating, estimates for π range from approximately 10 for below-BBB-rated debt to approximately 80 for AA-rated debt. Unfortunately, these estimates are upward biased because they include both liquidity and tax effects. We report results for each $\pi = \{1, 2, 5, 10\}$. (The case of $\pi = 1$ corresponds to no risk premium, meaning a deposit insurance premium equal to the estimated actuarial mean loss rate.)

5. Fair-premium estimates

This section uses our parameter estimates to obtain the FDIC fair-market insurance premiums based on (14). We report these insurance premiums for all surviving banks as of the second quarter of 2000 that meet the following screening criteria: (1) the bank must have at least eight quarters of data available prior to the second quarter of 2000, and (2) the bank must not have experienced more than 50% growth in any single quarter over the last five years, to mitigate the effect of mergers and acquisitions on our estimates. After these filters, 7401 banks remain in the sample.

Figure 2 illustrates the fair-market premium estimates for these 7401 banks for the second quarter of 2000, based on various assumptions regarding the fraction of insured deposits that would be recovered by the FDIC at failure. Specifically, for purposes of calculating the fair premium, we assume that the risk-neutral expected fractional recovery of assessed deposits at failure is the stated fractional recovery of insured deposits divided by $1 + \beta$, for a factor β which we set at 0.1, based on our analysis of the cross-sectional distribution of the ratio of uninsured to insured deposits that we collected from the call-

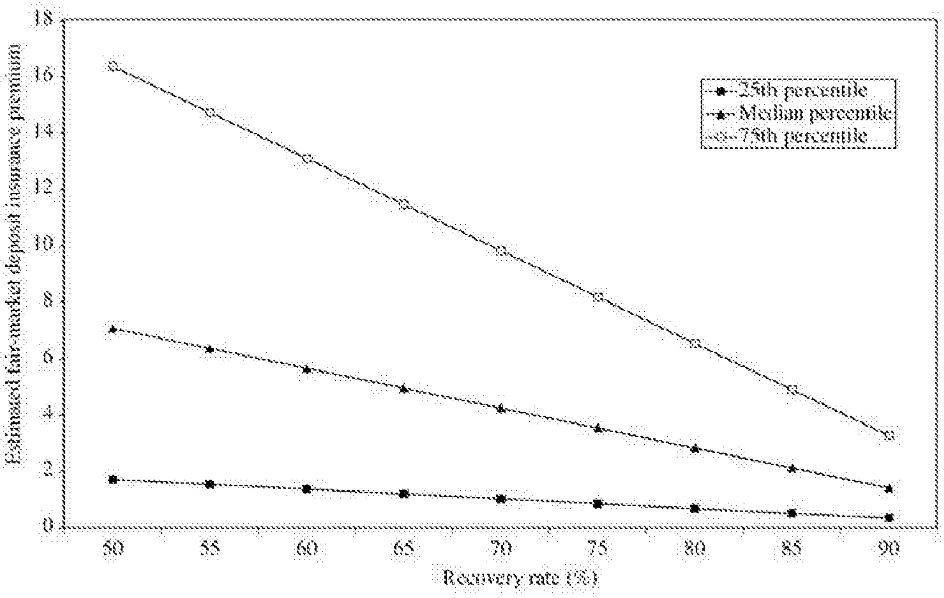


Figure 2. **Estimated fair-market insurance premia, dependence on recovery rates.** Fitted, annualized, fair-market deposit insurance premia for the second quarter of 2000, implied by a risk-premium factor of 5, for various alternative fractional rates of recovery of insured deposits, and for the median, upper-quartile, and lower-quartile levels of estimated fair-market deposit insurance premia. The risk-neutral expected fractional recovery of assessed deposits at failure is taken to be the indicated fractional recovery of insured deposits divided by 1.1. The risk-neutral mean failure rate is assumed to be five times the current actuarial (logit) hazard-rate estimate of the indicated quantile.

report data. (The cross-sectional median of this ratio was 8.8%, as we reported in section 2.)

The illustrated fair-market deposit premium calculation is for the second quarter of 2000, under the assumption that the risk-neutral mean failure rate is five times the current actuarial (logit) hazard-rate estimate. Table 6 provides results for other risk-premia scalings, and for other percentiles. Table 6 also shows the mean fair premium across the sample of banks, and the ''winsorized'' mean of the premium, which is the mean of the distribution that is obtained after removing from the sample the extreme 2% observations (1% each from both tails of the distribution of banks sorted by estimated failure hazard rate).

To illustrate further these fair-market premium estimates, table 7 provides the estimated fair-market deposit insurance premia for a selection of banks, based on an assumed risk-neutral expected fractional loss of total deposits of 27.3%. Also shown are the total deposits of these banks (in millions of dollars) as of the end of second quarter of year 2000. The predicted failure probability is also based on that estimated for the second quarter of 2000. The fair-market insurance premium is reported in basis points, that is, in cents per $100 of assessed deposits, annualized. The estimated deposit insurance premia are provided on an actuarial basis (no upward scaling for risk premia), and also under the

Table 6. **Estimated deposit insurance rates**

Recovery Rate (%)	Mean	Winsorized Mean	5th	10th	25th	Median	75th	90th	95th	99th
Panel A: Assuming risk-neutral intensity equal to empirical intensity										
50	2.98	2.76	0.01	0.04	0.35	1.54	4.04	7.75	10.49	18.43
55	2.68	2.49	0.01	0.03	0.31	1.38	3.63	6.98	9.44	16.58
60	2.38	2.21	0.00	0.03	0.28	1.23	3.23	6.20	8.39	14.74
65	2.08	1.93	0.00	0.03	0.24	1.08	2.83	5.43	7.34	12.90
70	1.79	1.66	0.00	0.02	0.21	0.92	2.42	4.65	6.29	11.06
75	1.49	1.38	0.00	0.02	0.17	0.77	2.02	3.88	5.25	9.21
80	1.19	1.10	0.00	0.01	0.14	0.62	1.62	3.10	4.20	7.37
85	0.89	0.83	0.00	0.01	0.11	0.46	1.21	2.33	3.15	5.53
90	0.60	0.55	0.00	0.01	0.07	0.31	0.81	1.55	2.10	3.69
Panel B: Assuming risk-neutral intensity is twice empirical intensity										
50	5.44	5.13	0.01	0.07	0.70	3.01	7.67	14.19	18.98	30.83
55	4.89	4.61	0.01	0.07	0.63	2.71	6.90	12.77	17.09	27.75
60	4.35	4.10	0.01	0.06	0.56	2.14	6.14	11.35	15.19	24.66
65	3.81	3.59	0.01	0.05	0.49	2.11	5.37	9.94	13.29	21.58
70	3.26	3.08	0.01	0.04	0.42	1.81	4.60	8.52	11.39	18.50
75	2.72	2.56	0.01	0.04	0.35	1.51	3.84	7.10	9.49	15.42
80	2.17	2.05	0.00	0.03	0.28	1.20	3.07	5.68	7.59	12.33
85	1.63	1.54	0.00	0.02	0.21	0.90	2.30	4.26	5.70	9.25
90	1.09	1.03	0.00	0.01	0.14	0.60	1.53	2.84	3.80	6.17
Panel C: Assuming risk-neutral intensity is five times empirical intensity										
50	10.96	10.55	0.03	0.18	1.72	7.09	16.38	27.65	35.21	50.37
55	9.87	9.49	0.02	0.16	1.55	6.38	14.71	24.88	31.69	45.33
60	8.77	8.44	0.02	0.14	1.38	5.67	13.10	22.12	28.17	40.29
65	7.67	7.38	0.02	0.13	1.20	4.96	11.46	19.35	24.65	35.26
70	6.58	6.33	0.02	0.11	1.03	4.25	9.83	16.59	21.13	30.22
75	5.48	5.27	0.01	0.09	0.86	3.54	8.19	13.82	17.60	25.18
80	4.39	4.22	0.01	0.07	0.69	2.83	6.55	11.06	14.08	20.15
85	3.29	3.16	0.01	0.05	0.52	2.13	4.91	8.29	10.56	15.11
90	2.19	2.11	0.01	0.04	0.34	1.42	3.28	5.53	7.04	10.07
Panel D: Assuming risk-neutral intensity is ten times empirical intensity										
50	16.76	16.32	0.05	0.36	3.36	12.80	25.97	38.95	47.31	64.14
55	15.08	14.68	0.05	0.32	3.03	11.52	23.37	35.05	42.58	57.72
60	13.41	13.05	0.04	0.29	2.69	10.24	29.78	31.16	37.85	51.31
65	11.73	11.42	0.04	0.25	2.36	8.96	18.18	27.27	33.12	44.90
70	10.05	9.79	0.03	0.21	2.02	7.68	15.58	23.37	28.39	38.48
75	8.38	8.16	0.03	0.18	1.68	6.40	12.98	19.48	23.66	32.04
80	6.70	6.53	0.02	0.14	1.35	5.12	10.39	15.58	18.92	25.66
85	5.03	4.90	0.02	0.11	1.01	3.84	7.79	11.69	14.19	19.24
90	3.35	3.26	0.01	0.07	0.67	2.56	5.19	7.79	9.46	12.83

assumption used in figure 2 of a risk-premium scaling of $\pi = 5$. Panel IA contains estimates for the banks that had not failed. Panel IB presents results for the banks that eventually failed during the second half of 2000 or in the year 2001–2002. Panel II provides the estimated Supervisory Sub-group and Capital Sub-group ratings of these

Table 7. **Estimated deposit insurance rates, selected banks**

Panel I: Modeled deposit insurance premia (in bp)

			Fair Insurance Premium	
	Deposits	Predicted Failure Prob.	$\pi = 1$	$\pi = 5$
Panel IA: Surviving banks				
Chase Manhattan Bank, NY	195,919.00	0.000050	0.037	0.182
Bank One NA, Chicago, IL	55,422.54	0.000032	0.038	0.190
Wachovia Bank NA, NC	43,218.90	0.000004	0.002	0.011
Fleet Bank NA, Jersey City, NJ	22,653.00	0.000027	0.023	0.117
Bank Midwest NA, Kansas City, MO	1344.32	0.000370	0.147	0.711
Bank One WV NA, Huntington, WV	1629.41	0.000310	0.241	1.169
Mechanics Bank, Richmond, CA	1314.79	0.000145	0.158	0.780
Capital Bank, Raleigh, NC	255.99	0.002659	0.156	0.760
Independence Bank, Owensboro, KY	137.54	0.005775	1.095	3.322
First State Bank of Fremont, NE	80.40	0.003848	4.180	14.729
Panel IB: Banks That Eventually Failed				
Oakwood Deposit Bank Co., Oakwood, OH	60.32	0.001600	1.615	6.915
The Bank of Honolulu, Hawaii	59.04	0.006850	17.615	50.214
First Alliance Bank & TC, Manchester, NH	21.49	0.081903	56.503	51.079
Bank of Sierra Blanca, Sierra Blanca, TX	12.37	0.105573	24.679	20.992
Malta National Bank, Malta, OH	7.41	0.007695	16.733	44.361

Panel II: Premium as per FDIC policy

	Capital Ratios					
Surviving banks	Supervisory Sub-Group	Total Cap. Ratio	Tier-1 Cap. Ratio	Tier-1 Lev. Cap. Ratio	Capital Sub-Group	Premium Rate as per FDIC
Chase Manhattan Bank, NY	A	11.56%	8.63%	5.70%	1	0
Bank One NA, Chicago, IL	A	12.37%	8.56%	6.86%	1	0
Wachovia Bank NA, NC	A	11.82%	7.75%	8.93%	1	0
Bank Midwest NA, Kansas City, MO	A	13.33%	12.07%	8.14%	1	0
Bank One WV NA, Huntington, WV	A	13.12%	11.87%	6.35%	1	0
Mechanics Bank, Richmond, CA	A	13.92%	12.73%	9.86%	1	0
Capital Bank, Raleigh, NC	A	11.18%	9.93%	8.23%	1	0
Independence Bank, Owensboro, KY	A	11.09%	9.90%	7.17%	1	0
First State Bank of Fremont, NE	A	11.58%	10.33%	7.02%	1	0

banks, as well as the associated current FDIC deposit insurance premia for these risk classifications. (As all these banks were rated 1A, the associated FDIC premia are 0.) Even assuming actuarial insurance rates, the FDIC appears to have been substantially subsidizing deposit insurance for several of these banks.

6. Illustrative valuation of future FDIC claims

We now illustrate the application of our modeling approach to the market valuation of long-term claims on the FDIC. Our affine state process X is driven by risk-neutral independent standard Brownian motions $B_1^*, B_2^*, \ldots, B_4^*$.

For this example, our dynamic model of the term structure of interest rates is the two-factor affine model of U.S. Libor interest rates that was used by Duffie et al. (2003) to analyze sovereign credit spreads. The parameters are maximum likelihood estimates based on weekly swap-rate observations from 1987–2000 at various maturities out to 10 years. For this model, the estimated risk-neutral[10] model of the short-term interest rate process is

$$r_t = 0.0503 + 0.000656X_{1t} + 0.000042X_{2t} \tag{15}$$

$$\mathrm{d}X_t = \begin{pmatrix} 0.0402 & 0 \\ -23.8 & 0.276 \end{pmatrix}\left[\begin{pmatrix} 12.4 \\ 0 \end{pmatrix} - X_t\right]\mathrm{d}t + \begin{pmatrix} \sqrt{X_{1t}}\mathrm{d}B_{1t}^* \\ \sqrt{1 + 3516X_{1t}}\mathrm{d}B_{2t}^* \end{pmatrix}. \tag{16}$$

We make the simplifying assumption that, at default, in risk-neutral expectation, bank debt loses a fraction ℓ_D of its face value, taking $\ell_D = 0.78$ as an estimate based. (Ammer and Packer (2000) estimated an average historical recovery at default rate of 78% on bank bonds, using Moody's data.) We further suppose that the bank has the risk-neutral default intensity process

$$\lambda^* = a + b_1X_1 + b_2X_2 + X_3,$$

where

$$\mathrm{d}X_{3t} = \kappa(\theta - X_{3t})\mathrm{d}t + \sigma\sqrt{X_{3t}}\mathrm{d}B_{3t}^*.$$

This allows for an easy calibration of the model to credit spreads at any given maturity. The parameters a, b_1, b_2, b_3, κ, θ, σ are calibrated so that the bank's short credit spread $S_t = \lambda_t^*\ell_D$ is initially 47 basis points, and mean reverts (under risk-neutral probabilities) to 94 basis points. That is, $\lim_{t\to\infty} E^*(S_t \mid \tau > t) = 94$ basis points. The parameters are also calibrated for a 20 basis-point increase in default intensity per 100 basis-point upward shift in interest rates. The motivation is the typical presence at banks of an asset–liability duration mismatch, under which an upward shift in the term structure of interest rates causes assets to be reduced in total market value more than liabilities, resulting in an effective increase in leverage and thus default risk. The precise parameter choices are otherwise somewhat arbitrary, and merely for illustration, and are listed in the Appendix.

For a deposit growth model, we take $D_t = \mathrm{e}^{X_4(t)}$, where

$$\mathrm{d}X_{4t} = (A + c_1X_{1t} + c_2X_{2t} + c_3X_{3t} + c_4X_{4t})\mathrm{d}t + \sigma_4\mathrm{d}B_{4t}^*,$$

10 Duffie et al. (2003) also provide estimates of the parameters determining interest-rate risk premia, and the associated model of the short rate under actual probabilities.

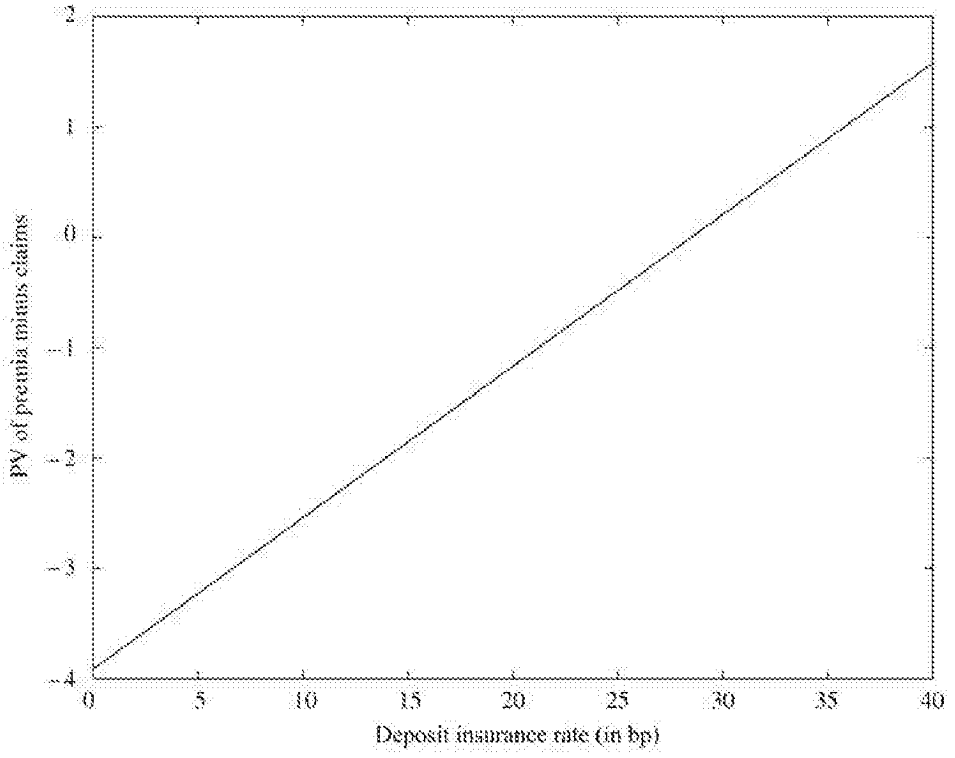
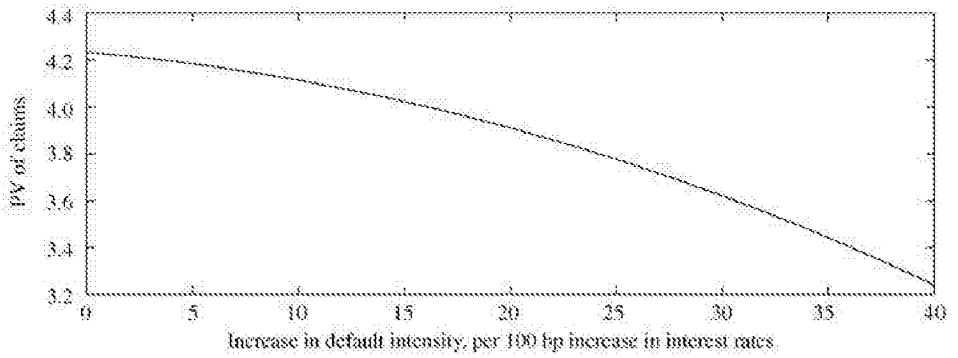


Figure 3. **Present value of 20-year-horizon claims net of premia at a constant premium rate, dollars per $100 of initial deposits.**

with constant parameters A, c_1, c_2, c_3, and c_4 are set for, under risk-neutral probabilities, a 50% rate of mean-reversion to deposit growth, and an expected decline in deposits of 5% per year per 100 basis-point increase in the short rate above its long-run mean. This is motivated by the idea that depositors move to money-market investments more aggressively when there is a wider spread between the interest rate offered on deposits and market interest rates. We also calibrate, again risk-neutrally, for an expected decline in deposits of 10% per year per 100 basis-point increase in the default intensity λ_t^* above its long-run mean, reflecting deposit runoff when the bank is under financial distress. The instantaneous volatility σ_4 of deposits was set at 10%. The precise parameter choices are listed in the Appendix. At bank failure, the FDIC is assumed to lose, in risk-neutral expectation, 30% of the last assessed deposits.

This model is a special case of the general affine state model of section 3, for the state vector $X = (X_1, X_2, X_3, X_4)'$.

For this hypothetical bank, the total present market value of the claims on the insurance provider, over a 20-year time horizon, is approximately $4 per $100 of initial deposits. This represents the capital that would be held by the FDIC to offset the market value of



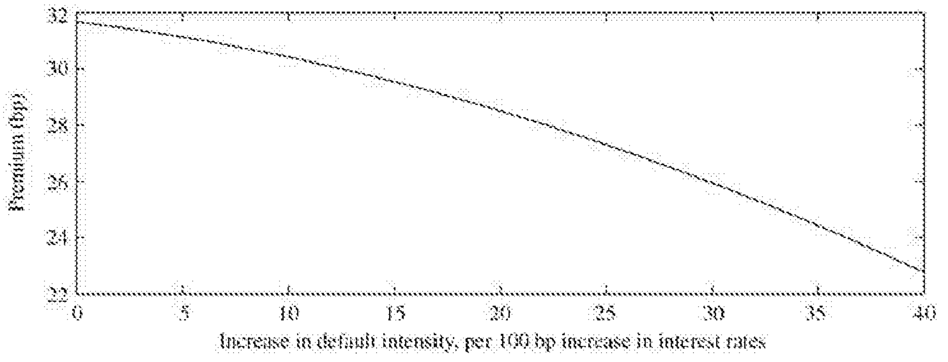


Figure 4. **Impact of sensitivity of credit risk to interest rates on the present market value of insurance claims, and on the 20-year fair-market premium.**

insurance claims over 20 years, before consideration of income from deposit insurance premia.

Of course, if the premium collected is adjusted at each contract period to the current fair-market premium level, then the net present value of long-horizon claims net of premia received is zero. In order to place the present value of future claims in perspective with alternative possible levels of deposit insurance premia, we consider the following experiment. Under the assumption that the deposit insurance premium rate is constant until failure, figure 3 illustrates the present value of the claims net of premia during this 20-year period, per $100 of initial deposits, for each of a range of constant premia. The ''break-even'' constant deposit-insurance premium for this illustrative example, that associated with zero present market value of future claims net of future premium payments, is approximately 27 basis points. In other words, for this model, 27 basis points would the fair-market premium for a contract period of 20 years (rather than the usual three-month contract period.) This happens to be the maximum premium rate that the FDIC currently charges.

Figure 4 shows how the present value of future claims, and the constant 20-year fair-market premium, depend on the degree of sensitivity of failure risk on interest rates. For example, if we double from its base case the sensitivity of the default hazard rate to the

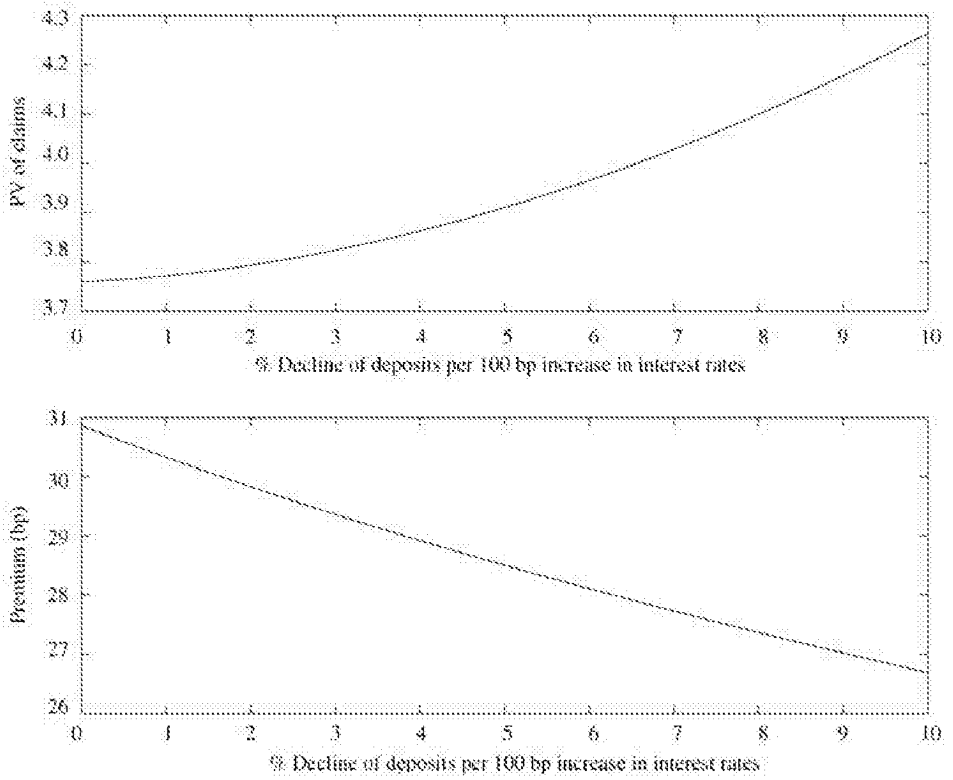


Figure 5. **Impact of sensitivity of deposits to market interest rates on the present market value of insurance claims, and on the 20-year fair-market premium.**

level of interest rates, the present value of 20-year claims on the insurer are reduced from roughly 4% of initial deposits to approximately 3.2%. This is due to a larger magnitude of negative correlation between discounts for interest rates and default losses. For example, a scenario in which interest rates go up significantly is associated with a likely increase in default losses, but losses in this scenario are more highly discounted, and thus have a smaller impact. This means that there are relatively fewer scenarios in which default losses are large and are carried to present value at adversely low interest rates. This represents a significant benefit to the FDIC in terms of the capital that it would hold to offset the present value of claims, assuming that its assigned insurance premia are below market rates.

Figure 5 shows, in a similar fashion, the role of sensitivity of deposit growth to interest rates. Scenarios in which interest rates decline are associated with two adverse effects for the insurer. Losses are less heavily discounted by interest rates, and losses are expected to be larger because deposits are expected to grow (because depositors have a smaller incentive to move away from bank deposits bearing low interest rates to other market-based investments). While this increased sensitivity of deposits to interest rates has an adverse effect, through correlation of deposits and interest rates, on the present value of future claims, it actually reduces the 20-year constant market-implied deposit insurance

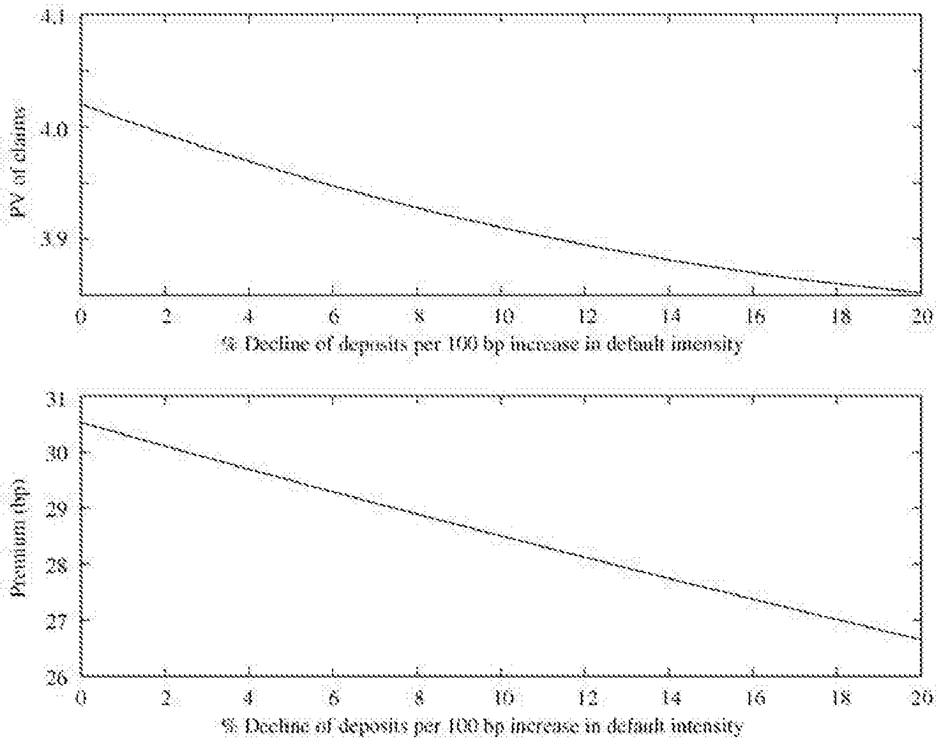


Figure 6. **Impact of sensitivity of deposits to bank credit risk on the present market value of insurance claims, and on the 20-year fair-market premium.**

premia. This is may appear, superficially, to be counterintuitive. The key point, however, is the large expected beneficial impact on the FDIC of a low interest rate scenario on the present value of premiums paid by the bank, because lower than expected interest rates are expected to be accompanied by lower than expected default hazard rates, and thus a longer expected stream of premiums. This beneficial correlation effect for the FDIC is even greater, in terms of the required fair-market premium rate, than the adverse correlation effect on the present value of claims.

Finally, figure 6 shows the impact of changing the sensitivity of deposit runoff to the bank's credit quality. As we increase the assumed sensitivity of depositors to the credit risk of their current bank, these depositors are more likely to reduce deposits as credit risk goes up. This correlation effect is beneficial to the insurer, because scenarios with high credit risk are associated with a low level of deposits to be covered in the event of failure.

7. Concluding remarks

This paper has presented a methodology and some empirical evidence regarding fair-market deposit insurance rates. We have restricted attention to the current form of FDIC insurance. Our approach is based on the reduced-form methodology that has recently become popular for analyzing and pricing credit risk. The evidence indicates that fair-market deposit insurance rates can be inferred from market pricing of the credit risk in bank debt instruments to be much larger than actuarially calculated mean loss rates to the insurer. Roughly speaking, the fair-market insurance rate of a given bank is the risk-neutral mean loss rates to the insurer, which is the product of (i) the annualized likelihood of failure during the period covered by the current contract and (ii) the expected loss to the insurer given failure, as a fraction of assessed deposits, both scaled to incorporate the effects of risk premia. An approximation of this deposit insurance rate is the product of (a) the bank's short-term credit spread and (b) the ratio of the insurer's expected loss at failure per dollar of assessed deposits to the bond investor's expected loss at failure per dollar of principal. For cases in which credit spreads for the bank or a comparable bank are not available, we have provided a methodology based on a logit model of the bank's actual default probability, scaled to allow for risk-premia.

Appendix

This appendix derives some analytical results used in section 3.1, provides the parameters used in section 6, and contains extra tables of results.

First, our objective is to calculate $\eta(s)$ of (12). Direct computation implies that

$$\begin{aligned}\eta(s) = \delta_0 + \delta_1 &\left[f(0,s) + \frac{1}{2} b(0,s)^2 \right] \\ &- \int_0^s f(0,u)\,\mathrm{d}u + \frac{1}{2}\int_0^s b(u,s)^2\,\mathrm{d}u \\ &+ \frac{1}{2}\left[\delta_1^2 \int_0^s \sigma_r^2 \mathrm{e}^{-2a(s-u)}\,\mathrm{d}u - \delta_1 b(0,s)^2 + \int_0^s b(u,s)^2\,\mathrm{d}u \right] \\ &+ \frac{1}{2}\delta_2^2\sigma_x^2 s,\end{aligned}$$

where $b(t,s) = \sigma_r(1 - \mathrm{e}^{-a(s-t)})/a$.

Let $\Gamma_0 = r_s = f(t,s) + b(t,s)^2/2 + \int_t^s \sigma_r \mathrm{e}^{-a(s-v)}\,\mathrm{d}B_1^*(v)$. We then let $\mu_0(t,s) = E_t^*(r_s) = f(t,s) + b(t,s)^2/2$ and let $\sigma_0^2(t,s) = \mathrm{var}_t^*(r_s) = \int_t^s [\sigma_r \mathrm{e}^{-a(s-v)}]^2 \mathrm{d}v$. Next, we let

$$\begin{aligned}\Gamma_1 &= \int_t^T r_s\,\mathrm{d}s \\ &= \int_t^T f(t,s)\,\mathrm{d}s + \int_t^T [b(t,s)^2/2]\,\mathrm{d}s + \int_t^T \int_t^s \sigma_r \mathrm{e}^{-a(s-v)}\,\mathrm{d}B_1^*(v)\mathrm{d}s.\end{aligned}$$

Letting $\mu_1(t,s) = E_t^*(\int_t^T r_s\,\mathrm{d}s)$, a direct computation gives

$$\mu_1(t,s) = \int_t^T f(t,s)\,\mathrm{d}s + \frac{1}{2}\int_t^T b(t,s)^2\,\mathrm{d}s.$$

Letting $\sigma_1^2(t,s) = \mathrm{var}_t^*(\int_t^T \tilde{\ } r_s\,\mathrm{d}s)$, we have $\sigma_1^2(t,s) = \int_t^s b(v,T)^2\,\mathrm{d}v$. Continuing, we let $\sigma_{01}(t,s) = E_t^*(r_s \int_t^s \tilde{\ } r_u\,\mathrm{d}u)$, and obtain $\sigma_{01}(t,s) = b(t,s)^2/2$.

Defining

$$\Gamma_2 \equiv G_s = G_t + \int_t^s \sigma_G\,\mathrm{d}B_2^*(v),$$

we have $\mu_2(t,s) \equiv E_t^*(G_s) = G_t$ and

$$\sigma_2^2(t,s) \equiv \mathrm{var}_t^*(G_s) = \sigma_G^2(s-t).$$

Given a bivariate normal pair (x,y) of random variables with respective risk-neutral means and standard deviations $(\mu_x, \sigma_x, \mu_y, \sigma_y, \sigma_{xy})$, we know that

$$E^*(\mathrm{e}^{Ax+By}) = \mathrm{e}^{\mu_x A + \mu_y B + [\sigma_x^2 A^2 + 2\sigma_{xy} AB + \sigma_y^2 B^2]/2}.$$

Now,

$$\begin{aligned} &E_t^*(\mathrm{e}^{\delta_1 r_s}\mathrm{e}^{-\int_t^s r_u \mathrm{d}u}\mathrm{e}^{\delta_2 G_s}) \\ &\quad = E_t^*(\mathrm{e}^{\delta_1 r_s}\mathrm{e}^{-\int_t^s r_u \mathrm{d}u})E_t^*(\mathrm{e}^{\delta_2 G_s}) \\ &\quad = \mathrm{e}^{\delta_1\mu_0(t,s) - \mu_1(t,s) + [\delta_1^2\sigma_0(t,s)^2 - 2\delta_1\sigma_{01}(t,s) + \sigma_1(t,s)^2]/2} \bullet \mathrm{e}^{\delta_2\mu_2(t,s) + \delta_2^2\sigma_2^2(t,s)/2}. \end{aligned}$$

The base-case parameters used for the example in Section 6 are

$a =$	-4.4×10^{-3}
$b_1 =$	3.58×10^{-4}
$b_2 =$	5.10×10^{-6}
$b_3 =$	1
$\kappa =$	0.5
$\theta =$	0.012
$\sigma =$	0.02
$A =$	0.205
$c_1 =$	-6.86×10^{-3}
$c_2 =$	-2.61×10^{-4}
$c_3 =$	-10
$c_4 =$	-0.5
$\sigma_4 =$	0.1

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